# Confidence Intervals for Population Means

**4/7/2020**

# Objective

By the end of the lesson, students should be able to…

1. Describe the sampling distribution being used
2. Know the assumption being made to conduct a 1 sample confidence interval
3. Apply the 1 sample confidence interval methods to a problem

# Review Problem #1

Which of the following are true statements?

1. Sample parameters are used to make inferences about population statistics.
2. Statistics from smaller samples have more variability.
3. Parameters are fixed, while statistics vary depending on which sample is chosen.

# Review Problem #2

Which of the following are true statements?

1. In all normal distributions, the mean and median are equal.
2. All bell-shaped curves are normal distributions for some choice of $\mu$ and $\sigma$.
3. Virtually all the area under a normal curve is within three standard deviations of the mean, no matter what the particular mean and standard deviation are.

# Answers

Review #1: 2 and 3 are correct. Sample statistics are used to estimate population parameters.

Review #2:1 and 3 are correct. Not all bell-shaped curves are normal curves. They can still be bell-shaped and not follow the empirical rule. The t-distribution is an example.

# Sampling Distribution for sample means

Center: the mean of all sample means is equal to the population mean. Thus, sample means are unbiased estimators of population means and can be used for inference.

Spread: the standard deviation is $\frac{\sigma}{\sqrt{n}}$. However, we rarely know $\sigma$, so we use $s_x$ instead. This is not a perfect estimate of σ, so we use the t-distribution to account for the error.

Shape: The central limit theorem applies here. We will address this on the next slide.

# Central Limit Theorem

When applying Central limit theorem. We know normal populations give normal sampling distributions. We also know that n>30, gives normal sampling distributions, but what about all the other cases? We will apply the following rule:

**We can consider the data approximately normal if...**

n<15, data roughly symmetric, single peak, no outliers

n≥15 no strong skew or outliers

n≥ 30

# Assumptions for 1 sample t-confidence intervals

In order to apply the one sample t-confidence interval, we need to make sure we meet three assumptions:

1. Random: The sample is from a random process, or the sample can fairly be considered representative
2. Independent: The process of sampling does not change the probability of the event happening, and if we are dealing with a finite population, that we sample less than 10% of the population.
3. Normal: We meet the one of the conditions of the central limit theorem. This might require making a graph.

# Example Problem

You have a batch of vitamin C pills and you would like to estimate the true mean amount of vitamin C per pill in the population. From a random sample of 16 pills, you find a mean of 517 mg and a standard deviation of 14mg. No outliers or strong skew were identified. Estimate the true population mean with 90% confidence.

# State

We wish to estimate the true population mean amount of vitamin C in the supplements.

$\mu$: the true mean amount of vitamin C in the pills

# Plan

We will use a 1 sample t-confidence interval with a 90% C.L.

Assumptions:

Random: The 16 pills are a random sample.

Independent: It is fair to assume that sampling one pill does not change the amount in other pills. It is also fair to assume that 16 pills is less than 10% of all Vitamin c pills.

Normal: Since the sample size is greater than 15 and we have no strong skew or outliers, we feel confident in assuming normality.

# DO

$\bar{x} = 517mg$

$s_x$ = 14 mg

n = 16

Confidence level = 90%

Degrees of freedom = 16-1 = 15 ← sample size minus 1

Critical value = 1.753 ← pulled from t-table with 15 df

Formula:

C.I. = $\bar{x} \pm t^*(\frac{sx}{\sqrt{n}}) \;=\; 517 \pm 1.753\,(\frac{14}{\sqrt{16}}) \;=\; 517 \pm 6.1355 \;\approx\; (510.86,\; 523.14)$

# Conclude

With 90% confidence, the interval 510.86mg to 523.14mg captures the true population mean milligrams of vitamin C in the pills.

# Common Mistakes

1. It is better to say the same thing twice, then forget to say it at all. Make sure to include everything. If you aren't sure you included it, write it again.
2. Do not simply show calculations state what they tell you.
3. Use the central limit theorem to determine normality of means… np>10 is for proportions only.
4. Remember the sample changes, the parameter is considered a constant. Never make statements implying the opposite.

# You try

To assess the accuracy of a laboratory scale, a standard weight that is known to weigh 1 gram is repeatedly weighed 4 times. The resulting measurements (in grams) are: 0.95, 1.02, 1.01, 0.98. Assume that the weighings by the scale when the true weight is 1 gram are normally distributed with mean $\mu$. Use these data to compute a 95% confidence interval for $\mu$.

# You try - answers

State: We wish to estimate the true mean measurement of a 1 gram weight by a laboratory scale.

$\mu$: the mean measurement of a 1 gram weight on the scale

# You try - answers

Plan: We calculate a 1 sample t interval with 95% confidence

Random: This is not by definition random, but the 4 measurements should be representative of the scale. We will proceed with caution.

Independent: It seems fair that one measurement will not affect the others. (This is not a finite population, so 10% condition does not apply)

Normal: With n=4, and a normal population distribution, we can assume normality.

# You try - answers

Do:

$\bar{x} = 0.99\ grams$

$s_x = 0.0316$ g

n = 4

Confidence level = 95%

Degrees of freedom = 4-1 = 3 ← sample size minus 1

Critical value = 3.182 ← pulled from t-table with 3 df

Formula:

C.I. = $\bar{x} \pm t^*(\frac{sx}{\sqrt{n}}) = 0.99 \pm 3.182\,(\frac{0.0316}{\sqrt{4}}) = 0.99 \pm 0.0503 \approx (0.9397,\ 1.0403)$

# You try - answers

Conclude: With 95% confidence, the interval 0.9397g to 1.0403g captures the true population mean measurement of the 1 gram weight.

(as a side note: since the interval includes the value 1, we should believe the scale is accurately measuring the weight. If it did not include 1 we would have evidence that something is wrong with the scale.)

# Extra examples and Practice

Reading: pg 499-516

HW: 49-52, 55, 57, 59, 63, 65, 67, 71, 73, 75-78